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Letter to the Editor

The Weakened First Algorithm of Remez

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The first algorithm of Remez is an algorithm for linear (real or complex) Chebyshev approximation on an infinite set which is a compact metric space. A convenient reference for the algorithm is Cheney [1, p. 96]. It is assumed henceforth that the reader is familiar with Cheney's notation. A difficulty with the algorithm is that the absolute maximum of the residual $r(c,x)=\sum c_i g_i(x)-f(x)$ must be located exactly, which is not possible numerically. We consider weakening this to selecting x^k, which is the maximum of $|r(c^k,\cdot)|$ on a (possibly irregularly shaped) grid of density $\leqslant \gamma_k$, where $\{\gamma_k\}\to 0$. We include the case where x^k is obtained by approximate uphill search from such a grid. We claim that the weakened algorithm still converges in the same way as the original. Let us now go to Cheney's proof. As $\{c^k\}$ is bounded, $\{r(c^k,\cdot)\}$ lies in a compact subset of $C(X)$ and so is equicontinuous, that is, given $M\delta>0$ there is $\eta>0$ such that

$$|r(c^k,x)-r(c^k,y)|<M\delta \qquad \text{if} \quad \rho(x,y)<\eta \text{ for all } k. \tag{$*$}$$

Now let $x^k_{\max}$ be the location of the absolute maximum of $r(c^k,\cdot)$ on X. Let $\hat{x}^k$ be the grid point closest to $x^k_{\max}$. Then

$$|r(c^k,\hat{x}^k)|\leqslant|r(c^k,x^k)|\leqslant|r(c^k,x^k_{\max})|.$$

Now select the index k in Cheney's proof to also have the density of the kth grid less than η satisfying $(*)$, then

$$||r(c^k,\hat{x}^k)|-|r(c^k,x^k_{\max})||<M\delta,$$

hence

$$||r(c^k,x^k)|-|r(c^k,x^k_{\max})||<M\delta.$$

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Now

$$\begin{aligned}\Delta(c^k) + M\delta &= |r(c^k, x^k_{\max})| + M\delta \\ &\leqslant |r(c^k, x^k)| + \big||r(c^k, x^k_{\max})| - |r(c^k, x^k)|\big| + M\delta \\ &\leqslant |r(c^k, x^k)| + M\delta + M\delta \\ &\leqslant |r(c^k, x^k)| + 2M\delta\end{aligned}$$

and the rest of the proof follows similarly to Cheney.

Note added in proof. Chapter 3 of the dissertation of D. H. Anderson, Australian National University, 1975, gives the first algorithm of Remez with search over a sequence of finite subsets whose density goes to zero. Anderson gives a convergence proof, some linear programming details, and a numerical example in two variables. Anderson did not consider an uphill search from the grid.

Reference

1. E. W. Cheney, "Introduction to Approximation Theory," McGraw–Hill, New York, 1966.

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